2224 – 7206M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2024

Mathematics: applications and interpretation

Higher level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) **EITHER**

$N = 72$		$N = 6$
$I = 1.25$		$I = 1.25$
$PV = -3000$	**OR**	$PV = -3000$
$P/Y = 12$		$P/Y = 1$
$C/Y = 12$		$C/Y = 12$

(M1)(A1)

Note: Award ***M1*** for attempt to use finance app on GDC, ***A1*** for all entries correct.

OR

$$3000\left(1+\frac{1.25}{1200}\right)^{72}$$ ***(M1)(A1)***

Note: Award ***M1*** for using CI formula, ***A1*** for correct entries.

THEN

$FV = (\$)\ 3233.53$ ***A1***

Note: Answer must be correct to 2 dp for the final ***A1*** to be awarded.

[3 marks]

(b) **EITHER**

$I = 1.25$		$I = 1.25$
$PV = -3000$		$PV = -3000$
$FV = 3550$	**OR**	$FV = 3550$
$P/Y = 12$		$P/Y = 1$
$C/Y = 12$		$C/Y = 12$

(A1)

Note: Award ***A1*** for all entries correct and opposite signs for PV and FV values.

OR

$$3550 = 3000\left(1+\frac{1.25}{1200}\right)^{12N}$$ ***(A1)***

Note: Award ***A1*** for all entries correct.

THEN

$N = 162\ (161.686\ldots)$ (months) ***A1***

[2 marks]

(c) 18450 ***A1***

[1 mark]

(d) **EITHER**

$N = 96$		$N = 96$
$I = 12.6$		$I = 1.05$
$PV = -18450$	**OR**	$PV = -18450$
$FV = 0$		$FV = 0$
$P/Y = 12$		$P/Y = 1$
$C/Y = 12$		$C/Y = 1$

(M1)(A1)

Note: Award ***M1*** for attempt to use finance app on GDC, ***A1*** for $N = 96$, $PV = -18450$ and $FV = 0$,

THEN

$PMT = (\$)\,306$ ***A1***

Note: The answer must be correct to the nearest dollar for the final ***A1*** to be awarded.

[3 marks]
[Total: 9 marks]

2. (a) (i) recognition of binomial distribution (condone incorrect parameter) ***(M1)***
e.g. $M \sim \mathrm{B}(20,\ 0.04)$ **OR** $\mathrm{P}(M = 2) = \mathrm{binpdf}(20,\ 0.04,\ 2)$
$= 0.146\ \ (0.145799...)$ ***A1***

(ii) recognition the cumulative probability required ***(M1)***
e.g. $\mathrm{P}(M \geq 3) = 1 - \mathrm{bincdf}(20,\ 0.04,\ 2)$ **OR** $\mathrm{bincdf}(20,\ 0.04,\ 3,\ 20)$
$= 0.0439\ \ (0.0438627...)$ ***A1***

[4 marks]

(b) either one of two terms in expected value formula correct ***(M1)***
$50\big(20(0.96)\big) + 15\big(20(0.04)\big)$
= 972 (pesos) ***A1***

[2 marks]
[Total: 6 marks]

3. (a) correct substitution of 0.0003 into the formula ***(A1)***

$\text{pH} = -\log_{10}(0.0003)$

$= 3.52 \quad (3.52287\ldots)$ ***A1***

[2 marks]

(b) **EITHER**

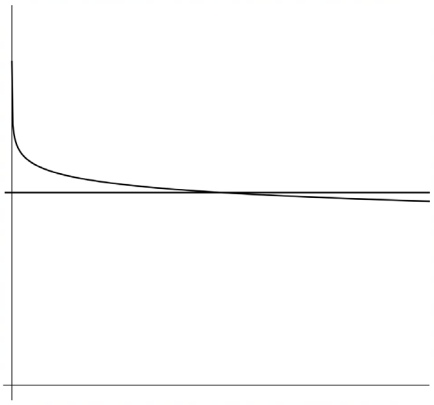
attempt to change to exponential form ***(M1)***

$\left[\text{H}^+\right] = 10^{-6.6}$

OR

attempt to solve $6.6 = -\log_{10}\left[\text{H}^+\right]$

(graphically or using numerical solver)



THEN

$\left[\text{H}^+\right] =$ 0.000000251 (moles per litre) $\quad (0.000000251188..., 2.51\times10^{-7})$ ***A1***

Note: Award ***M1A0*** for an answer of 2.51 $(2.51188...)$ seen.
Award ***M0A0*** if a substitution of 6.6 into the formula is seen without an answer or some indication of using numerical solver.

[2 marks]

(c) $2 = -\log_{10}\left[\text{H}^+\right],\ 4.5 = -\log_{10}\left[\text{H}^+\right]$

10^{-2} (0.01) **OR** $10^{-4.5}$ $(0.0000316227...)$ ***(A1)***

substitution of their values into correct ratio ***(M1)***

$\dfrac{10^{-2}}{10^{-4.5}}$ **OR** $\dfrac{0.01}{0.0000316227...}$

$= 316.227... = 316$ ***A1***

Note: Some candidates may subtract logs and hence look to solve $\log_{10}[\text{H}^+] = 2.5$.

[3 marks]

[Total: 7 marks]

4. (a) recognizing that only way to score 7 is to achieve a head and a 6 on die ***(M1)***

e.g. $\frac{1}{6}$ and $\frac{1}{2}$ seen in an attempt to combine probabilities

$\left(\frac{1}{6}\times\frac{1}{2}=\right)\ \frac{1}{12}\ (0.0833333...)$ ***A1***

Note: Condone 0.0835 from the use of 0.167.

[2 marks]

(b) there are two ways to score (e.g.) 5

achieve a head and a 4 on die, or a tail and a 5 on die ***(M1)***

$\left(2\left(\frac{1}{6}\times\frac{1}{2}\right)=\right)\ \frac{2}{12}\quad\left(\frac{1}{6}, 0.167, 0.16666...\right)$ ***A1***

Note: Award these marks for equivalent working for the 2, 3, 4 or 6 point scenarios.

Final Score	1	2	3	4	5	6	7
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$

A1

Note: Award ***A1*** for a completely correct table. Award at most ***(M1)A1A0*** if their follow-through answer from part (a) leads to a total probability not equal to 1.

[3 marks]

(c) **EITHER**

multiplying at least two columns from their table ***(M1)***

$1\times\frac{1}{12}+2\times\frac{1}{6}+...+6\times\frac{1}{6}+7\times\frac{1}{12}$

OR

recognizing the probabilities in the table are symmetric ***(M1)***

OR *(for HL markscheme and/or removed from SL at publication)*

Considering the sum of two random variables ***(M1)***

$\mathrm{E}(X+Y)=\mathrm{E}(X)+\mathrm{E}(Y)\ \ (=3.5+0.5)$

THEN

(expected value $=$) 4 ***A1***

Note: Accept $4.01\ (4.00640\ldots)$ from use of their 3 sf values from (b).
Award at most ***M1A0*** if their final answer is not in the range 1 – 7

[2 marks]

[Total: 7 marks]

5. (a) recognizing that the growth in one year is the difference in the two heights ***(M1)***

$5 \times 0.8 = 4 \text{ (m)}$ ***A1***

[2 marks]

(b) recognizing geometric sequence, $r = 0.8$ ***(M1)***

attempt to find total height by adding initial height to a term in series ***(M1)***

EITHER

$42 + \frac{4(1-(0.8)^6)}{1-(0.8)}$ ***(A1)***

OR

$37 + \frac{5(1-(0.8)^7)}{1-(0.8)}$ ***(A1)***

THEN

$= 56.7571\ldots$

$= 56.76 \text{ (m)}$ **OR** 5676 (cm) ***A1***

Note: Accept an answer in cm or in m to two decimal places.

[4 marks]

(c) attempt to use infinite geometric series to find the total growth of the tree ***(M1)***

e.g. $\frac{5}{1-0.8}$ **OR** $\frac{4}{1-0.8}$

$\left(S_\infty = 37 + \frac{5}{1-0.8}, S_\infty = 42 + \frac{4}{1-0.8} \right)$

$k = 62$ ***A1***

[2 marks]

[Total: 8 marks]

6. attempt to substitute into area of triangle formula ***(M1)***

(sheep's field area =) $0.5\times15\times21\times\sin(78°)$

$=154.058\ldots\ (\text{m}^2)$ ***A1***

EITHER

(goat's field area =) $\frac{282}{360}\times\pi\times8^2$ ***(A1)(A1)***

Note: Award ***A1*** for 282, ***A1*** for correct entries in formula (including their 282).

OR

$\pi\times8^2-\frac{78}{360}\times\pi\times8^2$ ***(M1)(A1)***

Note: Award ***A1*** for minor sector area, ***M1*** for subtracting their sector area from circle area.

THEN

$=157.498\ldots\left(\frac{752\pi}{15}\right)\ (\text{m}^2)$ ***A1***

the goat has most area by $3.44\ (\text{m}^2)\ (3.44026\ldots)$ ***A1***

Note: Accept 154 and 157 for the intermediate ***A1*** marks, but do NOT follow through within the question; a final answer of $3\ \text{m}^2$ is awarded ***A0***.

[Total: 6 marks]

7. attempt at Euler ***(M1)***

$y_{n+1}=(y_n x_n-1)0.1+y_n$ ***(A1)***

x	1	1.1	1.2	1.3	1.4	1.5
y	2	2.1	2.231	2.39872	2.61055…	2.87603…

(A1)

Note: Any correct value either as a separate calculation or as part of a table, (including the result).

$y=2.88\ (2.87603...)$ ***A1***

[Total: 4 marks]

8. (a) $\begin{pmatrix}1\\1\\2\end{pmatrix}$ ***A1***

Note: Accept any valid vector notation. Do not accept $(1, 1, 2)$.

[1 mark]

(b) (i) **EITHER**

use of scalar product formula to find angle $\mathrm{B\hat{A}D}$ ***(M1)***

$\cos \mathrm{B\hat{A}D} = \dfrac{\begin{pmatrix}3\\4\\0\end{pmatrix} \bullet \begin{pmatrix}1\\1\\2\end{pmatrix}}{5\sqrt{6}}$ ***(A1)***

OR

use of cosine rule to find $\mathrm{B\hat{A}D}$ ***(M1)***

$\cos \mathrm{B\hat{A}D} = \dfrac{6+25-17}{2\times\sqrt{6}\times 5}$ ***(A1)***

THEN

$\mathrm{B\hat{A}D} = 55.1^\circ \ \ (55.1417...^\circ, 0.962405...)$ ***A1***

Note: If the direction of one of the vectors is reversed, leading to an obtuse angle $(124.858...^\circ)$ between the vectors, then award ***M1A1A0***.

(ii) **EITHER**

an attempt at using vector product ***(M1)***

$\dfrac{1}{2}\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\times\begin{pmatrix}3\\4\\0\end{pmatrix}\right|$ or equivalent

attempt to apply formula (e.g. one correct component) ***(M1)***

$= \dfrac{1}{2}\left((1\times 0-4\times 2)\boldsymbol{i}-(1\times 0-2\times 3)\boldsymbol{j}-(1\times 4-1\times 3)\boldsymbol{k}\right)$

$\dfrac{1}{2}\left|\begin{pmatrix}-8\\6\\1\end{pmatrix}\right|$ ***(A1)***

$= 5.02 \ \left(5.02493, \dfrac{\sqrt{101}}{2}\right)$ ***A1***

OR

use of formula for the area of a triangle ***(M1)***

area = $\frac{1}{2}\left|\begin{pmatrix}3\\4\\0\end{pmatrix}\right|\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\right|\sin 55.1417\ldots$ ***(A1)(A1)***

$= 5.02$ ***A1***

Note: Award ***A1*** for the lengths AB and AD and ***A1*** for the angle.

[7 marks]

(c) **EITHER**

$\vec{\mathrm{AD}} = \vec{\mathrm{BC}} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ **OR** $\vec{\mathrm{AB}} = \vec{\mathrm{DC}} = \begin{pmatrix}3\\4\\0\end{pmatrix}$ ***A1***

One pair of opposite sides have equal length AND are parallel ***R1***

OR

$\vec{\mathrm{AD}} = \begin{pmatrix}1\\1\\2\end{pmatrix}, \vec{\mathrm{BC}} = \begin{pmatrix}1\\1\\2\end{pmatrix}, \vec{\mathrm{AB}} = \begin{pmatrix}3\\4\\0\end{pmatrix}, \vec{\mathrm{DC}} = \begin{pmatrix}3\\4\\0\end{pmatrix}$ ***A1***

$\vec{\mathrm{AD}} = \vec{\mathrm{BC}} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ and $\vec{\mathrm{AB}} = \vec{\mathrm{DC}} = \begin{pmatrix}3\\4\\0\end{pmatrix}$

both pairs of opposite sides are parallel (or have equal length) ***R1***

Note: Both pairs of opposite angles are equal is also valid.

THEN

hence ABCD is a parallelogram ***AG***

[2 marks]

[Total: 8 marks]

9. (a) (i) let X be number of accidents per week $\Rightarrow X \sim \text{Po}(0.76)$
recognition cumulative Poisson distribution ***(M1)***
$\text{P}(X \geq 2) = 1 - \text{P}(X \leq 1)$
$= 0.177 \ \ (0.176907...)$ ***A1***

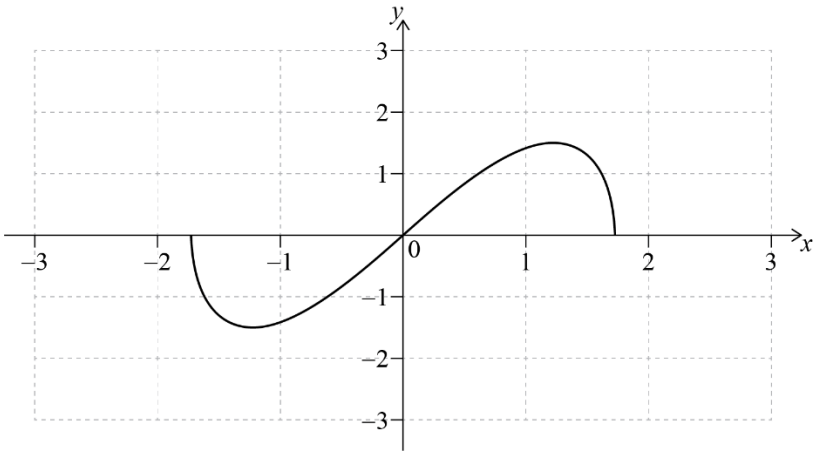
(ii) let F be number of accidents per 4-week $\Rightarrow F \sim \text{Po}(3.04)$ ***(M1)***
$\text{P}(Y = 3) = 0.224 \ \ (0.223982...)$ ***A1***

[4 marks]

(b) binomial model ***A1***
with $p = 0.177$ **OR** $p = 1 - 0.177 \left(= 0.823\right)$ and $n = 8$ ***A1***

[2 marks]
[Total: 6 marks]

10. (a)



A1A1

Note:	Award ***A1*** smooth curve, through origin, with maximum in 1st quadrant and minimum in 3rd quadrant (approximately correct position). Award ***A1*** correct (approximate) position of endpoints and rotational symmetry. Condone a (small) gap between end of curve and the x-axis.

[2 marks]

(b) (i) $2\pi\int_0^{\sqrt{3}} \left(x\sqrt{3-x^2}\right)^2 \, \text{d}x$ **OR** $\pi\int_{-\sqrt{3}}^{\sqrt{3}} \left(x\sqrt{3-x^2}\right)^2 \, \text{d}x$ (or equivalent) ***A1A1***

Note:	Award ***A1*** for integral with correct limits in the correct places, ***A1*** for completely correct expression; $\text{d}x$ must be included. Condone a correct answer presented as two integrals, despite the demand of the question: $\pi\left(\int_{-\sqrt{3}}^{0} \left(x\sqrt{3-x^2}\right)^2 \, \text{d}x + \int_0^{\sqrt{3}} \left(x\sqrt{3-x^2}\right)^2 \, \text{d}x\right)$.

(ii) $13.1 \left(13.0593..., \dfrac{12\pi\sqrt{3}}{5}\right)$ ***A2***

[4 marks]

(c) attempt to express $g(x)$ in terms of $f(x)$ ***(M1)***

$g(x)=\frac{1}{2}\left(\frac{x}{2}\sqrt{3-\left(\frac{x}{2}\right)^2}\right)$, $g(x)=\frac{1}{2}f\left(\frac{x}{2}\right)$

correct limits for g seen: $-2\sqrt{3}$ and $2\sqrt{3}$ ***(A1)***

$\pi\int_{-2\sqrt{3}}^{2\sqrt{3}}\left(\frac{1}{2}\left(\frac{x}{2}\sqrt{3-\left(\frac{x}{2}\right)^2}\right)\right)^2 \mathrm{d}x$ **OR** $\pi\int_{-2\sqrt{3}}^{2\sqrt{3}}\left(\frac{1}{2}f\left(\frac{x}{2}\right)\right)^2 \mathrm{d}x$ **OR** $2\times\frac{1}{2}^2\times\left(\frac{12\pi\sqrt{3}}{5}\right)$

$6.53\left(6.52967\ldots,\frac{6\pi\sqrt{3}}{5}\right)$ ***A1***

Note: Some candidates may answer question by transforming their part (b)(ii): $2\times\frac{1}{2}^2\times\left(\frac{12\pi\sqrt{3}}{5}\right)$. This is a valid method and leads to the correct answer.

[3 marks]

[Total: 9 marks]

11. (a) $y = kx^2 - x$

$\frac{dy}{dx} = 2kx - 1$ ***(A1)***

$m_{tan} = 1$ ***A1***

finding negative reciprocal of their gradient ***(M1)***

$m_{normal} = -1$

normal: $y = -\left(x - \frac{1}{k}\right)$ ***A1***

equating normal line to quadratic curve ***M1***

$kx^2 - x = -\left(x - \frac{1}{k}\right)$

correct simplification leading to given result ***A1***
e.g. quadratic formula or difference of two squares

$kx^2 - \frac{1}{k} = 0$

$k^2x^2 - 1 = 0$

$(kx - 1)(kx + 1) = 0$

$x = -\frac{1}{k}$ ***AG***

[6 marks]

(b) area $= \int_{-\frac{1}{2}}^{\frac{1}{2}} -\left(x - \frac{1}{2}\right) - (2x^2 - x)\ dx$ ***(M1)(A1)***

Note: Award ***M1*** for evidence of subtracting the functions and ***A1*** for a completely correct integral. (Condone a missing dx.)

$= 0.333\ \left(0.333333..., \frac{1}{3}\right)$ ***A1***

[3 marks]
[Total: 9 marks]

12. (a) $\sqrt{6^2+6^2+3^2}$ ***(A1)***

$=9\ (\text{m s}^{-1})$ ***A1***

[2 marks]

(b) $\boldsymbol{h}=\begin{pmatrix}-38\\134\\315\end{pmatrix}+t\begin{pmatrix}15\\-20\\-60\end{pmatrix}$ ***A1***

[1 mark]

(c) equating one component from each ***(M1)***

e.g. $7+6t=-38+15t$

$t=5$ ***(A1)***

substituting their t-value into either equation ***(M1)***

$\begin{pmatrix}-38\\134\\315\end{pmatrix}+5\begin{pmatrix}15\\-20\\-60\end{pmatrix}$

$=\begin{pmatrix}37\\34\\15\end{pmatrix}$ ***A1***

[4 marks]
[Total: 7 marks]

13. (a) $3\sin(a)(1+\cos(a))=0$ ***(M1)***

$a=3.14\ \ (\pi,\ 3.14159\ldots)$ ***A1***

[2 marks]

(b) attempt to find the maximum velocity ***(M1)***

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t}=0$ **OR** sketch of v with maximum marked

$v_{\max}=3.90(\text{m s}^{-1})\ \ (3.89711...)$ ***A1***

Note: Accept 3.9 rounded to 2 sf.

[2 marks]

(c) attempt to find integral of speed (condone omission of modulus signs) ***(M1)***

$\int_0^{\pi}\left|3\sin(t)(1+\cos(t))\right|\mathrm{d}t$ **OR** 6

attempt to substitute into $\text{speed}=\frac{\text{distance}}{\text{time}}$ ***(M1)***

$=\frac{\int_0^{\pi}\left|3\sin(t)(1+\cos(t))\right|\mathrm{d}t}{\pi}$ ***(A1)***

$=1.91\ (\text{ms}^{-1})\ \ (1.90985\ldots,\ \frac{6}{\pi})$ ***A1***

[4 marks]
[Total: 8 marks]

14. (a) room temperature / the temperature below which the hot water will not cool ***A1***

Note: Accept answers similar to "the temperature of the water in the cup after 25 minutes".

[1 mark]

(b) evidence of subtracting 25 from the temperature data ***(M1)***

$a = 244\ (243.920...)$ and $b = -1.03\left(-1.02965...\right)$ ***A1A1***

Note: Award ***M0A1A0*** for both answers $a = 116.764...$ and $b = -0.375218...$ seen, from not subtracting the 25.

[3 marks]

(c) $k = 61.1\ (61.0848...)$ and $c = 0.923\ (0.923029...)$ ***A1***

Note: If not subtracting 25 was penalized in part (b), award ***A1*** for $k = 68.9023...$ and $c = 0.972506...$ seen in part (c).

[1 mark]

(d) *Award **R1** for any appropriate reason that supports Soo Min's model*
e.g.
EITHER
compare r^2 values: $0.76573583\ldots$ and $0.9233117\ldots$ ***R1***

OR
consider value at $t = 0$ / "water cannot reach a temperature more than 100 degrees" ***R1***

Note: If subtracting 25 was penalized in (b) then award ***R1*** for comparing r^2 values $0.904086\ldots$ and $0.994522\ldots$

Award ***R1*** for correct reasoning seen for the use of comparison of the sum of square residuals.

[1 mark]
[Total: 8 marks]

15. (a) $\sqrt{2}\,e^{\frac{3\pi i}{4}}$ ***A1A1***

[2 marks]

(b) (i) enlargement SF $\sqrt{2}$, $(1.41,\ 1.41421\ldots)$, (centre $(0,\ 0)$) ***A1***

rotation of $\frac{3\pi}{4}$, $(2.36,\ 2.35619\ldots)$, (centre $(0,\ 0)$) ***A1***

(ii) $\begin{pmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{pmatrix}\begin{pmatrix}\cos\left(\frac{3\pi}{4}\right) & -\sin\left(\frac{3\pi}{4}\right)\\ \sin\left(\frac{3\pi}{4}\right) & \cos\left(\frac{3\pi}{4}\right)\end{pmatrix}$ ***(A1)(A1)***

Note: The order of matrix multiplication does not matter in this case.

$=\begin{pmatrix}-1 & -1\\ 1 & -1\end{pmatrix}$ ***A1***

Note: Do not ***FT*** if their matrices in part (b)(i) are not representative of a dilation AND a rotation.

[5 marks]

(c) 8 ***A1***

Note: Since $\frac{3\pi n}{4}$ must be a multiple of 2π.

[1 mark]
[Total: 8 marks]